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Students' perception of letters as specific unknown and variable: The Case of Iran

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ABSTRACT
The purpose of this study was to assess student's understanding of variable concept. In respect to the changes of math curriculum in Iran, especially way that variable was introduced in the new textbook, it is necessary to assess students' understanding of variable involving these new approaches. The present study followed an explanatory sequential mixed method design. The subjects of the study were 7th, 8th and 9th grade students living in different area of Tehran city. Data was collected from a written test and semi structured interviews. Our sample was 400 students. Interviews conducted with 15out of them. By analysing data and textbook based on Kuchemann'(1981) and Sfard's(1991) theories, students with such training were not recognizing the role of variable in different situation and often see it as specific unknown. They also made mistakes for simplifying of algebraic expression and they tried to get solution by joining of numbers and letters.

Keywords: Algebra; Algebraic expressions; Students' perception; Kuchemann's model; Variable

INTRODUCTION

Algebra has vital role in mathematics and its role as a gatekeeper, make it important for learning and teaching of mathematics (Knuth et.al. 2005) algebra is a generalized form of arithmetic. The use of letters and signs make algebra as an abstract subject so it is considered to be a difficult area of mathematics (Samo, 2009). It is also viewed as the study of regularities governing numerical relation, a conception that centers on generalization and that can be widened by including the components of proof and validation. (Bednarz, Kieran & Lee, 1996).

Variable concept is an essential tool in teaching and learning school mathematics and so as in other fields (such as: physics, chemistry, economy and ……). Therefor understanding the variable concept is the basis to all abstract concepts in mathematics.

Leitzel (1989) states that understanding the concept of variable is so essential that, difficulty in variable concept would make the students' fail in algebra. The concept of variable is an important concept for transition from arithmetic to algebra. Student's difficulties in variable concept can lead to more difficulties in higher level mathematics, solving algebraic equations, etc.

Understanding variable concept provides a link between arithmetic and algebra (Ursini & Trigueros, 2001). There are differences in arithmetic and algebra such as: equations solving and calculations. As calculations in arithmetic are executed on numbers, while algebra requires reasoning over the quantitative unknowns, or variables.in spite of these differences, Yet Van Ameron (2003) states that there is also an interrelated link between arithmetic and algebra as: algebra is performed on the arithmetical operators and furthermore, in some cases, arithmetical equations act algebraically.

Regarding importance of variable role in school mathematics, results of many researchers conducted in the area of learning and teaching algebra indicate that; Students have difficulties in understanding symbol/letters (Kuchemann, 1981; Macgregor & Stacy, 1997; Dede, 2004; Akgun & Ozdemir, 2006).

Usiskin (1988) defines that majority of students recognize all variables as letters which are an abbreviation for numbers, whereas variables do not always represent numbers. For example, in geometry, variables represent points, as in triangle "ABC", a line "AB", so on. In analyses, variables are abbreviations representing words, for

instance f represents function. In linear algebra, variable A is an abbreviation for the Matrix and variable v is an abbreviation for the Vectors.

Therefore much of the difficulties of students in variable concept, may be the result of their inability in recognizing the real role of literal symbol (Philipp, 1992), since used same letters in different contexts with different meaning or may be used the same letters in similar situations, with different meanings, cause problems in conceptual understanding of the concepts of algebra (Zahid, 1998).

Regarding students difficulties understanding variable concept, students' confrontation method with variable concept and introduction technique of this concept has specific importance in school textbook mathematics.

Iranian mathematics curriculum was changed in 2009. The objective of curriculum modification had been: Establishing connection within mathematics and daily life, acquiring skills in modeling mathematics and calculation solution, developing thinking skills and …. On the other hand social developments and changes, mathematical teaching developments, and new teaching approaches has triggered changes in students' needs and expectations, thus this new curriculum, has considered students and society's new expectations.

Textbook contents are considered to be essential elements in education curriculum. Our country's educational policy type is centralization model, and in most cases textbook contents are the only teaching tools for the teachers. Therefore teaching-learning process is executed merely based on the concepts and values laid down. In present study, researchers' emphasis is based on new textbook contents since, after establishment of new curriculum, modality of implementing new curriculum and though preparing textbook contents has been the major concern for authors.

In new textbook, variable has been introduced in 7^{th} grade for the first time. Efforts have been made to improve students' understanding of variables concept to help them transition from arithmetic to algebra. In respect to the changes of mathematics curriculum in Iran, and especially the way the concept of variable was first introduced in the new textbook, it is necessary to assess the students' understandings of this concept involving these new approaches. This study aims to assess student's perception on variable, and in particular to answer the following questions:

How do students of 7^{th}, 8^{th} and 9^{th} grades, understand the variable concept, in relation to the changes made in the math textbooks? Does their understanding improve through grades?

THEORETICAL FRAMEWORK

In the present study, two theories of Kuchemann(1981) and Sfard (1991) were used and data analysed based on these theories.

Kuchemann's categorization of variables

Kuchemann's study (1981) in concept of Secondary Mathematics and Science (CSMS) project, investigated learners interpretation of letters. His study was based on categories which were developed by Collis (1975). Evaluation was conducted with a sample of Ten thousand students ranging from 11 to 15 years old. He identified six different categories. Categories of this study are as follows:

Letter evaluated: this category applies to responses where a numerical value is assigned to letter from the outset. In other words, this category refers to items where children are asked to find a specific value for an unknown but again without first having to operate on the unknown.

Letter not used: Here the children ignore the letter, or at the best acknowledge its existence without giving it a meaning.

Letter used as an object: The letter is regarded as shorthand for an object or as an object in its own right.

For the first three interpretations, the letters are evaluated arithmetically and not as unknown. However, in the next three interpretations the letter needs to be interpreted algebraically, given meaning and operations need to be performed directly on the letter.

Letter used as a specific unknown: Children regard the letter as a specific but unknown number, and can operate on it directly.

Letter used as a generalized number: The letter is seen as representing, or at least as being able to take several values rather than just one.

Letter used as variable: The letter is seen as representing a range of unspecified values, and a systematic relationship is seen to exist between such set of values. In this interpretation the letter can take a range of values

which seems similar to the interpretation as generalized number, but 'the concept of a variable implies an understanding of an unknown as its value changes' (Kuchemann, 1981).

Kuchemann (1981) determined the level of understanding by two dimensions, which are the interpretation of letters and the structural complexity of tasks. At the two lower level tasks (level 1 and 2), the all items can be solved without having to operate on letters representing numbers. In fact the items were purely numerical and had a simple structure or the letters can be evaluated, regarded as **objects**, or **not used**. In contrast, for items at level 3 and 4, the letters have to be treated at least as **specific unknowns** or **generalized numbers** and in some cases as **variables**. Difference between level 1 and 2 and also level 3 and level 4 can be described in terms of item complexity.

Table 1 presents the six uses of variables with their definitions as documented by Kuchemann (1981) with examples.

Table 1: Six uses of variables documented by Kuchemann (1981)

Role of letter	Definition	Example
Letter evaluated	Numerical value that can be determined by trial and error	$x + 6 = 12\ , x =?$
Letter ignored	No need to handle the expression containing the variable	$a + b = 6\ , a + b + 2 =?$
Letter as object	Shorthand for an object rather than for a characteristic of it	$p = e + e + a$ 
Letter as specific unknown	Specific, abeit unknown, number that can be operated on without evaluating	$a + b = 6\ , a + b + c =?$
Letter as generalized number	Multiple values can be taken	$x + y < 20\ , x > y\ , y =?$
Letter as variable	Relationship between letter as their value systematically changes	$x + 2$ یا 2xبزگتر است

Sfard's theory of reification

Sfard (1991) distinguishes between operational and structural conception of the same mathematical notions. According to Sfard, if a learner, interpreting a notion as a process, implies regarding it as a potential rather than actual entity, and in contrast, another learner seeing a mathematical entity as an object, indicates being capable of referring to it as if it was a real thing and also being able to manipulate it as a whole, without going into details. The former is called operational conception and the latter is structural conception.

According to Sfard's theory, transition from operational to abstract objects is a long and inherently difficult process, accomplished in three steps: interiorization, condensation and reification. She explains : " first there must be a process performed on the objects already familiar (interiorization), then the idea of turning this process into an autonomous entity should emerge (condensation) and finally the ability to see this new entity as an integrated, object- like- whole must be acquired (reification)."

Sfard (1991) states that reification and structural conception step is essential for conceiving more complex and higher level processes. This means that different processes should be converted into a whole condensed static so that the new object could also be a basis for higher levels, and if we consider this model of constructing mathematical concepts as whole, it is conceived hierarchically. That is, one level must be understood and accomplished to begin the next level, and this process-object-process cycle ..... can be repeated frequently so that complex mathematical concepts can be constructed.

For example, the counting operation results in the concept of number and the concept of number becomes as a basic object for conceiving the concepts of addition and multiplication, power, decimals, and fraction and Correspondingly proceeds to advanced levels mathematics. In each of these cases, there is a cycle of conceptualization in the construction of particular mathematical concepts.

LITERATURES

Many researches have been conducted related to students' misconceptions of variables. In addition to five staged that delineated by Kuchemann (1981) that represented common misconceptions of students in interpreting letters, further common misconceptions have been seen in literatures as: misconception is seen when students treat variables as a label for an object (McNeil et al., 2010), thinking that two different variables (like x, y) in the same equation cannot represent the same value (Swan, 2000), assigning numerical value to

letters according to its position in the alphabet (Asquith et al.2007, Herscovics &Kieran, 1980) and failing to understand its value changes(Stacey & MacGregor,2000; Stephens ,2005).

There are lots of studies showing that students' misconceptions of algebra are mostly related with their lack of knowledge about the arithmetical operations (Mulungye, 2010; Norton & Irvin 2007; Stacey & Chick, 2004). Some problems may occur when the calculations are offered as algebraic expressions. The calculations are still named as subtraction, addition, multiplication, and division but sometimes it is observed that students cannot be serial in doing operations as they are in arithmetical expressions (Jupri & Drijvers, 2016; Uçar Sarımanoğlu, 2019). Besides, their emphasis on computation leads to many misconceptions in students' minds, which in turn makes the learning of algebra become more difficult (Uçar Sarımanoğlu, 2019).

Wagner's (1983) contribution to the concept of variable

Wagner (1983) added an interesting dimension to the discussion of variable by explaining ways in which variables are similar and different from numbers and words.

He explained that; on one hand, variables are like words in which they act as placeholders. On the other hand, variables are different from words because, although they can mean differently in different contexts, they must maintain the same meaning throughout the same context, whereas words can change meanings even within the same sentence. Another distinction between variables and words which Wagner provided was that, while words are made up of letters, there are specific rules about the way they are used together as opposed to the use of letters for variables (Dogbey, 2010).

Phillip (1992) categorization of letters

Philipp (1999) also presented another arrangement of variable usage which was divided into 7 categories and then set an example for each to illustrate the usage. They were as follows;

Letters as labels: f and y in to denote 3 feet in 1 yard;
Letters as constants: π, e and c
Letters as unknowns: denote x in
Letters as generalized numbers: denote a, b in
Letters as varying quantities: denote x, y in
Letters as parameters: denote m, b in
Letters as abstract symbols: denote e, x in

Variable concept in Iranian mathematic textbook

Regarding that the textbook is the main teaching tool in our country, accordingly existing educational approaches into textbook contents will have effect on teachers' educational strategies. Therefor prior to analysing student's perception on variable concept, it is so required that; new mathematics textbook structure and so on variable introduction method in 7^{th} grade textbook be explained and delineated.

New mathematics curriculum has been designed with special attention to the early algebra approach. The algebraic concepts have been implicitly considered from the second to the sixth level of elementary education, but the algebraic concept of variable is clearly the goal of training in the seventh grade. Newly written textbooks for the new mathematics curriculum are structured in such a way that, each lesson begins with student-centred activities, which, according to planners, are meant to teach the concepts, followed by class activities and exercises to strengthen and consolidation and deepening the concept. The educational program for teaching the variable concept is presented in chapter 3 of mathematics textbook for 7th grade students. In the first section of seventh grade's mathematic textbook, the concept of the variable is presented by using a numerical pattern. The main goal of this section is to expand the concept of letter to the concept of variable. In order to do this, three activities with different contexts are designed for introduction of the variable concept.

In the first activity, the context is about a square-shaped wrestling mat, which aims to build a functional relationship between two variables, size for the mats side and the Perimeter. In this activity, with the different values assigned to the mattress side variable, students are required to determine perimeter and vice versa.

In the second activity, a geometric pattern is given and students are asked to complete the table according to the figure number and the number of circles in each shape.

In the third activity, in the same way, there is a geometric pattern and a number of matches have been drawn in each shape, and more detailed questions have been asked from students so that at the end of questions, students can guess the relationship between the figure number and the number of matches.

At the end of these activities, the lesson on variable ends with the following definition:

In this activity, the circumference of the square with side a is $4 * a = 4a$. The letter a is called variable. In algebra, these symbols are used to indicate unknown numbers or unspecific values.

After this, some class activities and exercises are presented in order to consolidate (based on curriculum' claim) the variable concept. Here, variety of representation patterns such as geometric representation, data table, and numerical representation are used. At the end of the chapter there is an activity which exemplifies that the arithmetic rules can be rewritten in algebraic expressions by using variables. The example has been illustrated that in addition, the commutative rule can be represented as: $a + b = b + a$.

After completing these activities, the first section which introduces the variable concept ends. The next chapter introduces the algebraic expressions.

Analysing textbook activities based on Sfard's model

Analysing variable concept's activity based on Sfard's model shows that in the first activity, with the allocation of certain amounts to the square's sides, the interiorization process of the Sfard model (1991) occurs. Because the student deals with tangible numbers, each time a certain amount is attributed to the size of the wrestling mat sides. By finding different values for the mattress perimeter consecutively and by calculating merely three values for the size of the mattress side, it is therefore expected that the replacement of the mattress side in a particular rule will condense (Sfard, 1991) a variable concept in the student's mind. Then by presenting the symbolic form of the variable a, it is expected that reification step of (Sfard, 1991) the variable concept to be formed. According to Sfard (1991), this is very difficult. To form the third stage of conceptualization, it is expected, in the first stage, that several tasks to be performed to prepare the ground for object conception to the concept of variable.

For the second time, the same trend has been followed in the second activity that it is not described in a specific context and promising mathematical model.

In the third activity, in which the relationship between the figure number and the number of matches is considered, after the student is asked to calculate the number of matches in 10 steps, finally is questioned to make a guess about the relationship between the figure number and the number of matches. As have been said, this is the stage of Sfard condensation (1991), which in the first two activities no attempt is made to formulates this important subject, and the textbook expects to be recognized on its own. Finally, in the third activity, the calculation of the number of matches for the n^{th} figure is asked, which seems to be the case in this activity.

According to Sfard (1991), for the realization and construction of a variable concept, initially it is necessary to begin the process with distinct numbers. After repeating a series of successive sequences with distinct numbers, the student will have a more general look at the relationship between the two variables (for example, in the previous activity, the relationship between the figure number and the number of matches). Of course, it is better to ask students questions in order to force the students to pay attention to this relationship and viewpoint, such as:

•How many matches are there in the 10th figure?

•How many matches are there in the 20th figure?

•How many matches are there in the 100th figure?

•Can you express the relationship between the figure number and the number of match sticks verbally?

By presenting such questions, at this stage, an operational schema emerges from the algorithms in the student's mind; in this stage, focus is merely on the relationship between the two variables (for example, on the figure number and the number of matches), symbols and signs, and ultimately, in the final stage, letters are replaced by tangible and familiar objects, in which the student understands the new concept by communicating between the components and their relationships. Therefore, in the conceptual design model of Sfard (1991), shifting the focus from operation and process to objects and thinking of these operations as objects is possible.

By analysing of construction of variable concept in 7^{th} grade textbook, based on Sfars theory, students with such training do not have a proper understanding of reification and entity of variable concept.

METHOD

This study is part of a large project. The purpose of this study was to gain insight into learners' interpretation of letters usage as specific unknown and variable. The present study followed an explanatory sequential mixed method design, to gather both quantitative and qualitative data (Creswell, 2014). The subjects of this study were 7th, 8th and 9th grade students of Tehran studying during 2016-2017 academic years.

Reason for selecting middle-grade schools for this study is that, middle-grade schools are vital stage for transition from arithmetic to algebra which, at this stage, students are more subjected to the variety of variable uses.

Samples were selected by cluster sampling method. Tehran was divided into five clusters, North, South, East, West and Central. Next a municipality district was randomly selected from each cluster. Afterwards from each chosen municipality district, one school for girls was selected which added up to five schools in all.

Due to existence of cultural restrictions and limitations in IRAN, girls and Boys schools are divided. Since the researcher as a female has only access to girls' school, therefore sample schools were selected from girl's schools.

Finally, from these 5 candidate schools selected, students from 7th, 8th and 9th grade were randomly chosen, which added up to 460 students in total. Later on 60 students were eliminated as a result of absentee on exam day and so as not giving answers to all questions in the exam. So final number of sample students covering this study had been 400 students in total.

DATA COLLECTION

Data was collected through implementing a written test with chosen sample students, and semi structured interviews went through with some of them (found appropriate for our study).

Kuchemann's (1981) framework was used as a lens for viewing the data collected. Data was analyzed using descriptive statistics. The CSMS (1979) test consists of 27 questions. But in this report, results of some questions related directly to usage of letter as a specific unknown and so as variable have been reported. Questions such as 4e, 4c and 7d, were designed to assess students' interpretations of letter as a specific unknown, and questions such as 3 were designed to assess students' interpretations of letter as variable.

Out of 400 students who participated in written tests, 15 students were chosen for interviews. These students included ones who answered correctly and so had routine procedure in their answers; and some of them had deep misconception and/or extraordinary answers.

Interviews were conducted for the minimum of 25-30 minutes. In the present study, semi-structured interview was a great research tool which supplemented the written tests, and was a key to trace and identify the points which seemed to be unclear and therefor had been missed in the minds of students in the written test. In fact, it was then used to get a deeper understanding of students reasoning for the response they had given to written test.

RESULTS

In this paper, out of 27 questions of CSMS test and 6 categories of letters interpretation of Kuchemann's framework (1981), two categories of letter as specific unknown and letter as variable have been used for analysis.

Letter as specific unknown: In this category, students interpreted the letter as specific but unknown number and performed operations on the letters.

Questions 4e & 4c of CSMS test: The frequency percentage of correct answers to Kuchemann's levels of CSMS test (questions 4c and 4e), by the students of 7th, 8th and 9th grades is given in Table 2 and Figure 1.

Table 2: Frequency percentage of students of 7th, 8th and 9th grades to 4e and 4c questions of CSMS test

Questions	Grade 7	Grade 8	Grade 9	Levels of kuchemann(1981)
4c: "Add 4 onto 3n"	28%	50%	57%	Level 3
4e: " multiply n+5 by 4"	1%	8%	33%	Level 4

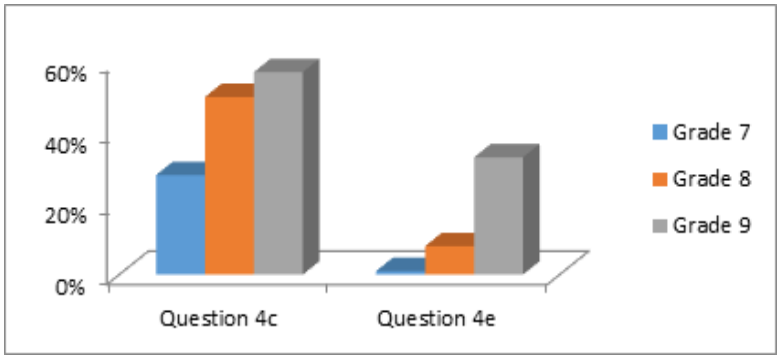

Figure 1: Student's answers to questions 4c and 4e of CSMS test

As seen in Table 2, only 28% of 7th grade students had answered correctly to question 4c, and about half of the 8th and 9th grade students did answer correctly to the same question. Students with incorrect answer to question 4c, did have misconception over the simplicity of algebraic expressions, and most common erroneous answer was 7n or sometimes wrote 7.

Question 4e was at level 4 of Kuchemann's framework, so in respect to questions in other levels is much more complicated. It was different from the rest of the questions in the interpretation since multiplication was used in the question. Only 1% of the seventh grade and 8% of eighth grade students answered correctly to this question.

As seen in Figure 1,with increase of the educational grades, the process of understanding the structural of algebraic expressions in the students increased and students could better determined structure and solve questions 4e and 4c of CSMS test.

Relative to the 7th grade students, students of 8th and 9th grade were more likely focused on multiplication and distribution property.

Common misconceptions and erroneous answers of students to this question are given in Figure 2.

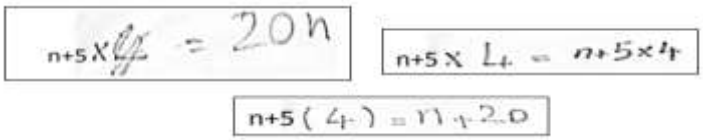

Figure 2: Common erroneous answers of students to question 4e of CSMS test

As is noticeable in answers, students in reference to their previous experience in arithmetic were mostly trying to get the result of an algebraic expression and getting a single answer. Though with no attention paid on to simplifying algebraic expressions, they are joined up to get to a single set, Which can be told that students in algebraic expressions mostly have desire to think arithmetically.

Questions 7d of CSMS test

Another question that related to the usage of letter as specific unknown and noticed as being so difficult to students was question 7d of CSMS test (Figure 3).

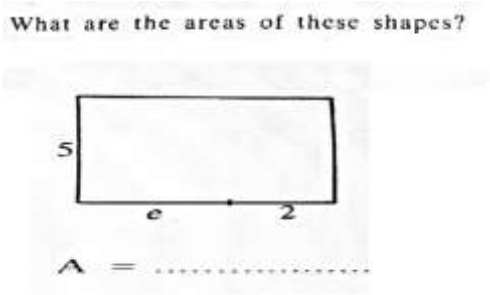

Figure 3: Question 7d of CSMS test

The frequency distribution of 7th, 8th and 9th grade students' answers to question 7d of CSMS test is given in Table 3 and Figure 4.

Table 3: Frequency distribution of 7th, 8th and 9th grade students' answers to 7d questions of CSMS test

	Frequency of 7th grade	Percentage	Frequency of 8th grade	Percentage	Frequency of 9th grade	Percentage
Correct answers	30	21%	44	29%	32	31%
Incorrect answers	105	73%	91	60%	63	59%
No answers	9	6%	15	11%	11	10%
Total	144	100%	150	100%	106	100%

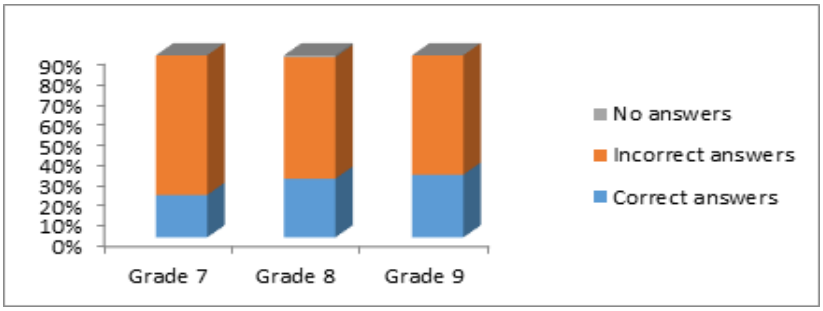

Figure 4: Student's answers to question 7d of CSMS test

As the results shown in table 3, incorrect answers percentage to this question surpasses the correct answers percentage in all academic grades, so majority of students have made error on question 7d. But with increasing of educational grades, percentage of correct answered increased.

In the next section some sample interview-scripts are given which supports these results. The common erroneous answers given to this question by students are shown in Figure 5.

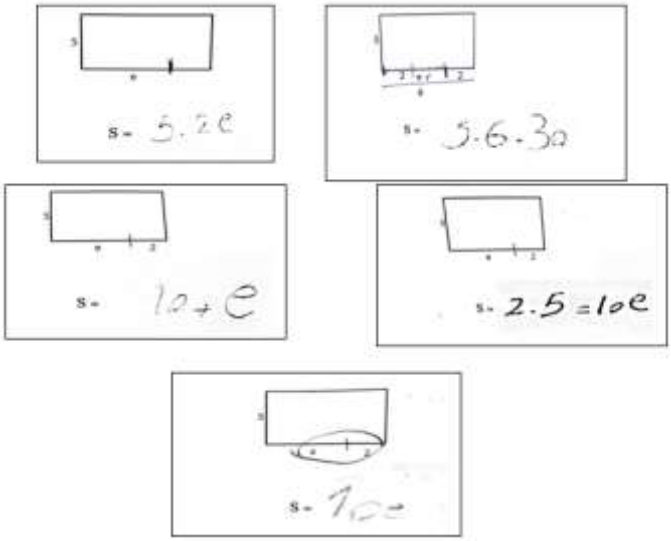

Figure 5: Common erroneous answers of students to question 7d of CSMS test

Interviews were conducted with students who had answered incorrectly to this question. Some scripts are as follows:

T: What is the area of this shape?

S: Area for rectangle is multiplication of length by width. Rectangle width is **5**, but the length is in two parts, one part is **2** and other part **e**, so the length adds up to **2e**.

T: well, then what will be the area?

S: length multiplied by width, so **5 × 2e = 10e**

Another student's answer to same question was as follows:

S: the area of the rectangle is multiplication of length by the width so that results in: **5 × 2 =10** and **5 × e = 5e** and final result is **15e**
T: how did you get **15e**?
S: I added up two parts together: **10 + 5e = 15e**

As the interviews indicate, students see the letter " e " as an object or an independent entity from the multiplication following which they can do the arithmetic expression without regarding the variable " e ", and finally place the letter " e " inside the answer. For example in 5e term, 5 is considered separated from" e "and as a result 10 is added up by 5 and" e "has been placed besides the 15. And still the error on getting a simple answer in interviews is visible.

Letter as variable
In this category, students distinguished between the ideas of a letter taking several values in turn and a letter representing a set of values simultaneously. The notion of "simultaneous values" leads to the concept of a variable (kuchemann, 1981).

Question 3 of CSMS test
In this question students were asked to answer which is larger, "2n" or "n+2". And afterwards were told to explain the answer given.
The frequency distribution of students' answers to question 3 of CSMS test is given in Table 4 and Figure 6.

Table 4: Frequency distribution of students' answers to question 3 of CSMS test

Question	Grade 7	Grade 8	Grade 9	Levels of kuchemann(1981)
Which is the larger, $2n$ or $n+2$? Explain:	2%	0%	12%	Level 4

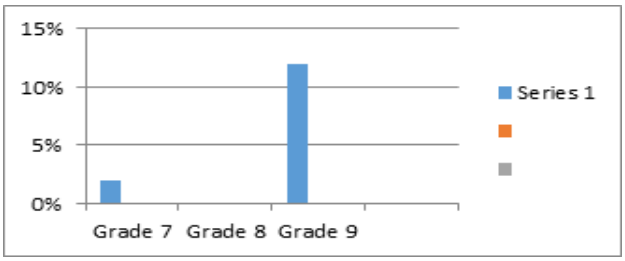

Figure 6: Student's answers to question 3 of CSMS test

As seen in table 4, a few of 7th (2%) and 9th (12%) grade students were able to attain the correct answer and none of the 8th grade students responded correctly to this question (0%). With increasing educational grades from 7th grade to 8th grade, understanding of students of variable, decreased but from 8th to 9th grade, the percentage of correct answered to this question, increased. So 9th grade students understand variable as "varying quantity " than other students. So it looks obvious that this question has been too complex for students. This question was one of the most difficult ones in the written test. Most common erroneous answer to this question was 2n.
Interviews were conducted with students who had answered incorrectly to the question. Sample interviews are as follows:
T: which one is larger, "**2n**" or "**n+2**"?
S: "**2n**"
T: Why?
S: it is clear, in "**2n**", "**n**" is multiplied by "**2**" but in "**n+2**", "**n**" is summed with **2**, and multiplications result is always larger than the sum.
Another students' response to this question in the interview is as follows:
Student: depends on the sign of "**n**".

Teacher: would you explain that?
Student: if "**n**" is positive, "**2n**" becomes larger, but if "**n**" is negative, then "**2n**" becomes smaller. Since the larger the negative number gets, lower the value becomes.
The common erroneous answers of students to this question are given in Figure 7.

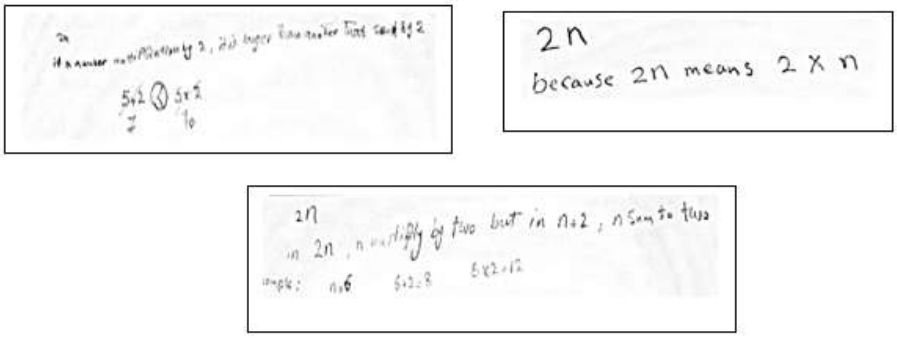

Figure 7: Common erroneous answers of students to question 3 of CSMS test

From the interviews and sample incorrect responses of students it's evident that they possessed minor perception of variable concept. Responses show that students can not recognize that n can retain different values, so they come to a misunderstanding in which n can only hold one specific value, not more than one value. Therefor upon giving one value or two, they come to a conclusion that either 2n is larger or 2+n.
Answering this question for the students requires that they compare every two sets together, with similar variable used in. So when comparing two sets, they ought to realize that n can retain a range of different values.
Students' answers indicate that, they may have substituted n=1 and n=2 values, yet since the results were differed with n>2, therefor as a result they overlooked these two values. And reference to n>2, had answered that 2n is larger.
By giving two or three numeric values to n, and presenting response over their results it is so definite that students mostly have desire to think arithmetically rather than algebraically.
Students which in their response have stated that: "multiplication will result in value being larger". Simply with no attention to variable "n", the operators have compared without paying attention to that, the objective is comparison of two terms which these operators have been used in.
Majority of students had considered n as being natural number. A few number of them had seen n either as being negative number. Then again for answering comparison between two designated expressions in the question had compared 2n by itself; "since when n is positive or n is negative, which they concluded that as if n is negative, 2n becomes smaller in value.
Most students in response to this question, simply by considering one value to n had come to a decision in which, either 2n is larger or 2+n, some of the students too, by giving two or three values to "n", had written their answers.
If students had recognized letter n as a variable (changing their values), could have simply answered this question accurately.

CONCLUSIONS

Variable is an important concept in mathematics and particularly in algebra which requires a rich concept image (Tall& Vinner, 1981). Therefore deep understanding of concept is not easy. Students in middle schools must have a conceptual understanding of various variable applications (NCTM, 2000). So it's necessary to furnish variety of opportunities for students dealing with various applications of variables.
Student's response to the CSMS test showed that they didn't know really the meaning of variable as a varying quantity and were unable to recognize different use of letters in a context. Furthermore the results of student's interviews clearly indicated that students experience great difficulty in the category of letter as specific unknown according to Kucheman's model. Generally results showed that they could not distinguish between letter as specific unknown and as variable. Once they see a letter in algebraic expressions consider it as a specific unknown.

Findings of present study showed that most of students were trying to get a single answer of algebraic expression as their prior experiences in arithmetic. They try to simplify algebraic expressions by joining of numbers and letters. For example most common erroneous answer of algebraic expression like "4+3n" was 7 o 7n.

Furthermore students had common error answer in the questions that involved multiplication like as "4(n+5)" or "5(e+2)".Most students try to solve these expressions arithmetically without regarding to the letters. In these expressions, there is multiplication distribution. The students perform multiplication operator on numbers and then they place letters next to the numbers. For example common erroneous answers of expression like "4(n+5)" were 20n. Generally students see the letters as independent entity from multiplication and try to solve algebraic expressions arithmetically.

In the present study, students showed the poorest performance on letters comprehension as variable. While a letter stand as variable, mostly they consider it as specific unknown and assign some familiar numbers in it. Therefore could not understand its changing nature and not assign numerous values to the letter. For example, results of student's interviews was indicated that in question 3 of CSMS test, most of students by specifying a few certain value of " n", and sometime with respect to multiplication operator, determined that always " $2n$ "is greater than" $n+2$ "

Despite of changing the Iranian mathematics curriculum and efforts to facilitate understanding of variable concept, we still see that students have a poor understanding of letters as variable. one of the reasons of students' misunderstanding in variable concept could be related to textbook content when it start to introduced this concept. And so on teaching strategies used by teachers, which definitely is affected by the textbook contents, which of course it was not the researchers' aim. Another reason for students' misconceptions and difficulties in algebra concepts especially in variable concept might be the poor definition of concept.

Mathematical definition has an important role in teaching and learning of mathematics (Ertekin, Yazici, & Delice, 2015). Students may, in life experiences or before entering school, be familiar with informal definitions but in textbooks they are encountered with formal definitions which develop and enrich their concept image. One of the ways to detect how students define a mathematical concept is to analyze how they have understood and use the concept.

Definition of a concept is its' written model in textbooks which of course is enhanced and fixed by activities and practices in their minds (Ulusoy, 2013).

As it is emphasized by the United States' Common Core State Standards for Mathematics [CCSSM] (Common Core State Standards Initiative, 2010), it is essential that basic concepts of algebra are defined correctly in order to understand the meaning of concepts (Long & DeTemple, 2003). Definition offered for variable concept in the textbook causes misunderstanding in students as it has introduced variable as unknown or as specific unknown. Definitely with this definition visual image created in students would be variable as a specific unknown, not as a varying quantity.

According to Kushman's theoretical framework, this definition is at Levels 4 and 5 of Kushman's theory, and by this definition, students do not reach to the level 6; that is letters conception as variables. It seems with this definition given in textbook, teachers' strategies emphasizing on difference between unknown and variable in students' mind will become so difficult. It is not surprising that students conceptualize the variables as unknowns since the mathematics curriculum in Iran defined variables as unknowns and So no wonder that this test results showed that majority of students see variable as specific unknown not as varying quantity.

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